

# Go forth, and Multiply!

Maths Club

Elliott Tjia

# Base instincts

- ▶ What is a number base?

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- ▶ What is our main number base?

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- ▶ What is our main number base?
  - ▶ Ten (Decimal)
  - ▶ Digits used: 0,1,2,3,4,5,6,7,8,9

# Base instincts

- ▶ What is a number base?
- ▶ What is our main number base?
  - ▶ Ten (Decimal)
  - ▶ Digits used: 0,1,2,3,4,5,6,7,8,9
- ▶ A number base is just a representation of the same information, and doesn't change the data itself.

# One, Two, Many, Many-One,...

- ▶ How did people count?

# One, Two, Many, Many-One,...

- ▶ How did people count?
  - ▶ Babylonians, base 60 counting system.
  - ▶ Mayans, base 20 - today is 13.0.0.3.4

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𐀆 2	𐀇 12	𐀈 22	𐀉 32	𐀊 42	𐀋 52
𐀌 3	𐀍 13	𐀎 23	𐀏 33	𐀐 43	𐀑 53
𐀒 4	𐀓 14	𐀔 24	𐀕 34	𐀖 44	𐀗 54
𐀘 5	𐀙 15	𐀚 25	𐀛 35	𐀜 45	𐀝 55
𐀞 6	𐀟 16	𐀠 26	𐀡 36	𐀢 46	𐀣 56
𐀤 7	𐀥 17	𐀦 27	𐀧 37	𐀨 47	𐀩 57
𐀪 8	𐀫 18	𐀬 28	𐀭 38	𐀮 48	𐀯 58
𐀰 9	𐀱 19	𐀲 29	𐀳 39	𐀴 49	𐀵 59
𐀶 10	𐀷 20	𐀸 30	𐀹 40	𐀺 50	

- ▶ Do we use any non-base ten counting?

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- ▶ Do we use any non-base ten counting?
  - ▶ Seconds in a minute, minutes in an hour, hours in a day

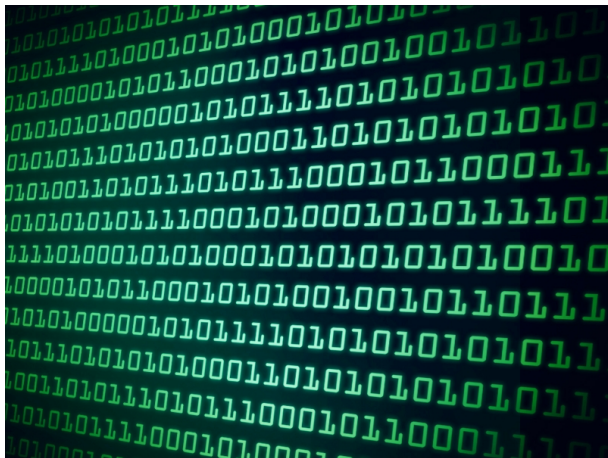


# An On-off relationship with computers

- ▶ How to Computers count?

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- ▶ How to Computers count?
- ▶ Base 2 (Binary)



# All your base are belong to us

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- ▶ Problem: Every base is base 10
- ▶ Notation for bases  $34_5 = 19_{10} = 10011_2$

# Something doesn't add up

- ▶ Addition and subtraction still work in the same fashion
- ▶  $33_7 - 12_7 =$

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- ▶  $33_7 - 12_7 = 21_7$
- ▶  $23_5 + 14_5 =$



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- ▶ Question Sheet 1

# My base is bigger than your base

- ▶ Hexadecimal is base 16
- ▶  $10_{10} = ??_{16}$

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- ▶ Hexadecimal is base 16
- ▶  $10_{10} = ??_{16}$
- ▶ Remembering that a base is defined by the number of unique digits used, base 16 can use the following digits:  
0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F

# By the Power of Grayskull

- ▶ Conversion between bases is easier if one base is an integer power of the other e.g.  $16 = 2^4$

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- ▶  $3_{16} = 11_2$
- ▶  $A_{16} = 1010_2$
- ▶  $3A_{16} = 111010_2$

- ▶ Classical Algorithm
  - ▶ Napier's bones
- ▶ Single Digit Additions, Multiplications, Shifts.



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  - ▶ Peasant Multiplication
- ▶ Karatsuba Algorithm
- ▶ Question Sheet 2

# Jack Skellington would be proud

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- ▶ Base 10 is Decimal

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- ▶ OCT 31=

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- ▶ Base 8 is Octal
- ▶ Base 10 is Decimal
- ▶ OCT 31=DEC 25